CHAPTER VII.

DISCUSSION OF THE DATA OF STATURE.

Stature as a subject for inquiry.—Marriage Selection.—Issue of unlike Parents.—Description of the Tables of Stature. Mid-Stature of the Population.—Variability of the Population.—Variability of Mid-Parents.—Variability in Co-Fraternities.—Regression: *a*, Filial; *b*, Mid-Parental; *c*, Parental; *d*, Fraternal.—Squadrons of Statures.—Successive Generations of a People.—Natural Selection.—Variability in Fraternities.—Trustworthiness of the Constants.—General view of Kinship.—Separate Contribution from each Ancestor.—Pedigree Moths.

Stature as a Subject for Inquiry.—The first of these inquiries into the laws of human heredity deals with hereditary Stature, which is an excellent subject for statistics. Some of its merits are obvious enough, such as the ease and frequency with which it may be measured, its practical constancy during thirty-five or forty years of middle life, its comparatively small dependence upon differences of bringing up, and its inconsiderable influence on the rate of mortality. Other advantages which are not equally obvious are equally great. One of these is due to the fact that human stature is not a simple element, but a sum of the accumulated lengths or thicknesses of more than a hundred bodily parts, each so distinct from the rest as to have earned a name by which it can be specified. The list includes about fifty separate bones, situated in the skull, the spine, the pelvis, the two legs, and in the two ankles and feet. The bones in both the lower limbs have to be counted, because the Stature depends upon their average length. The two cartilages interposed between adjacent bones, wherever there is a movable joint, and the single cartilage in other cases, are rather more numerous than the bones themselves. The fleshy parts of the scalp of the head and of the soles of the feet conclude the list Account should also be taken of the shape and set of the many bones which conduce to a more or less arched instep, straight back, or high head. I noticed in the skeleton of O'Brien, the Irish giant, at the College of Surgeons, which is the tallest skeleton in any English museum, that his great stature of about 7 feet 7 inches would have been a trifle increased if the faces of his dorsal vertebræ had been more parallel than they are, and his back consequently straighter.

This multiplicity of elements, whose variations are to some degree independent of one another, some tending to lengthen the total stature, others to shorten it, corresponds to an equal number of sets of rows of pins in the apparatus Fig. 7, p. 63, by which the cause of variability was illustrated. The larger the number of these variable elements, the more nearly does the variability of their sum assume a "Normal" character, though the approximation increases only as the square root of their number. The beautiful regularity in the Statures of a population, whenever they are statistically marshalled in the order of their heights, is due to the number of variable and quasi-independent elements of which Stature is the sum.

Marriage Selection.—Whatever may be the sexual preferences for similarity or for contrast, I find little indication in the average results obtained from a fairly large number of cases, of any single measurable personal peculiarity, whether it be stature, temper, eye-colour, or artistic tastes, in influencing marriage selection to a notable degree. Nor is this extraordinary, for though people may fall in love for trifles, marriage is a serious act, usually determined by the concurrence of numerous motives. Therefore we could hardly expect either shortness or tallness, darkness or lightness in complexion, or any other single quality, to have in the long run a large separate influence.

I was certainly surprised to find how imperceptible was the influence that even good and bad Temper seemed to exert on marriage selection. A list was made (see Appendix D) of the observed frequency of marriages between persons of each of the various classes of Temper, in a group of 111 couples, and I calculated what would have been the relative frequency of intermarriages between persons of the various classes, if the same number of males and females had been paired at random. The result showed that the observed list agreed closely with the calculated list, and therefore that these observations gave no evidence of discriminative selection in respect to Temper. The good-tempered husbands were 46 per cent. in number, and, between them, they married 22 good-tempered and 24 bad-tempered wives; whereas calculation, having regard to the relative proportions of good and bad Temper in the two sexes, gave the numbers as 25 and 21. Again, the bad-tempered husbands, who were 54 per cent. in number, married 31 good-tempered and 23 bad-tempered wives, whereas calculation gave the number as 30 and 24. This rough summary is a just expression of the results arrived at by a more minute analysis, which is described in the Appendix, and need not be repeated here.

Similarly as regards Eye-Colour. If we analyse the marriages between the 78 couples whose eye-colours are described in Chapter VIII., and compare the observed results with those calculated on the supposition that Eye-Colour has no influence whatever in marriage selection, the two lists will be found to be much alike. Thus where both of the parents have eyes of the same colour, whether they be light, or hazel, or dark, the percentage results are almost identical, being 37, 3, and 8 as observed, against 37, 2, and 7 calculated. Where one parent is hazel-eyed and the other dark-eyed, the marriages are as 5 observed against 7 calculated. But the results run much less well together in the other two possible combinations, for where one parent is light and the other hazel-eyed, they give 23 observed against 15 calculated; and where one parent is light and the other dark-eyed, they give 24 observed against 32 calculated.

The effect of Artistic Taste on marriage selection is discussed in Chapter X., and this also is shown to be small. The influence on the race of Bias in Marriage Selection will be discussed in that chapter.

I have taken much trouble at different times to determine whether Stature plays any sensible part in marriage selection. I am not yet prepared to offer complete results, but shall confine my remarks for the present to the particular cases with which we are now concerned. The shrewdest test is to proceed under the guidance of Problem 2, page 68. I find the Q of Stature among the male population to be 1·7 inch, and similarly for the transmuted statures of the female population. Consequently if the men and (transmuted) women married at random so far as stature was concerned, the Q in a group of couples, each couple consisting of a pair of *summed* statures, would be $\sqrt{2} \times 1·7$ inches $= 2·41$ inches. Therefore the Q in a group of which each element is the *mean* stature of a couple, would be half that amount, or 1·20 inch. This closely corresponds to what I derived from the data contained in the first and in the last column but one of Table 11. The word "Mid-Parent," in the headings to those columns, expresses an ideal person of composite sex, whose Stature is half way between the Stature of the father and the transmuted Stature of the mother. I therefore conclude that marriage selection does not pay such regard to Stature, as deserves being taken into account in the cases with which we are concerned.

I tried the question in another but ruder way, by dividing (see Table 9) the male and female parents respectively into three nearly equal groups, of tall, medium, and short. It was impracticable to make them precisely equal, on account of the roughness with which the measurements were recorded, so I framed rules that seemed best adapted to the case. Consequently the numbers of the tall and short proved to be only approximately and not exactly equal, and the two together were only approximately equal to the medium cases. The final results were :—32 instances where one parent was short and the other tall, and 27 where both were short or both were tall. In other words, there were 32 cases of contrast in marriage, to 27 cases of likeness. I do not regard this difference as of consequence, because the numbers are small, and because a slight change in the limiting values assigned to shortness and tallness, would have a sensible effect upon the result. I am therefore content to ignore it, and to regard the Statures of married folk just as if their choice in marriage had been wholly independent of stature. The importance of this supposition in facilitating calculation will be appreciated as we proceed.

Issue of Unlike Parents.—We will next discuss the question whether the Stature of the issue of unlike parents betrays any notable evidence of their unlikeness, or whether the peculiarities of the children do not rather depend on the *average* of two values; one the Stature of the father, and the other the transmuted Stature of the mother; in other words, on the Stature of that ideal personage to whom we have already been introduced under the name of a Mid-Parent. Stature has already been spoken of as a well-marked instance of the heritages that blend freely in the course of hereditary transmission. It now becomes necessary to substantiate the statement, because it is proposed to trace the relationship between the Mid-Parent and the Son. It would not be possible to discuss the relationship between either parent singly, and the son, in a trustworthy way, without the help of a much larger number of observations than are now at my disposal. They ought to be numerous enough to give good assurance that the cases of tall and short, among the unknown parents, shall neutralise one another; otherwise the uncertainty of the stature of the unknown parent would make the results uncertain to a serious degree. I am heartily glad that I shall be able fully to justify the method of dealing with Mid-Parentages instead of with single Parents.

The evidence is as follows :—If the Stature of children depends only upon the *average* Stature of their two Parents, that of the mother having been first transmuted, it will make no difference in a Fraternity whether one of the Parents was tall and the other short, or whether they were alike in Stature. But if some children resemble one Parent in Stature and others resemble the other, the Fraternity will be more diverse when their Parents had differed in Stature than when they were alike. We easily acquaint ourselves with the facts by separating a considerable number of Fraternities into two contrasted groups: (*a*) those who are the progeny of Like Parents; (*b*) those who are the progeny of Unlike Parents. Next we write the statures of the individuals in each Fraternity under the form of $\mathsf{M}+(\pm D)$ (see page 51), where M is the mean stature of the Fraternity, and D is the deviation of any one of its members from M. Then we marshal all the values of D that belong to the group *a*, into one Scheme of deviations, and all those that belong to the group *b* into another Scheme, and we find the Q of each. If it should be the same, then there is no greater diversity in the *a* Group than there is in the *b* Group, and such proves to be the case. I applied the test (see Table 10) to a total of 525 children, and found that they were no more diverse in the one case than in the other. I therefore conclude that we have only to look to the Stature of the Mid-Parent, and need not care whether the Parents are or are not unlike one another.

The advantages of Stature as a subject from which the simple laws of heredity may be studied, will now be well appreciated. It is nearly constant in the same adult, it is frequently measured and recorded; its discussion need not be entangled with considerations of marriage selection. It is sufficient to consider the Stature of the Mid-Parent and not those of the two Parents separately. Its variability is Normal, so that much use may be made of the curious properties of the law of Frequency of Error in cross-testing the several conclusions, and I may add that in all cases they have borne the test successfully.

The only drawback to the use of Stature in statistical inquiries, is its small variability, one half of the population differing less than 1·7 inch from the average of all of them. In other words, its Q is only 1·7 inch.

Description of the Tables of Stature.—I have arranged and discussed my materials in a great variety of ways, to guard against rash conclusions, but do not think it necessary to trouble the reader with more than a few Tables, which afford sufficient material to determine the more important constants in the formulæ that will be used.

Table 11, R.F.F., refers to the relation between the Mid-Parent and his (or should we say *its?*) Sons and Transmuted Daughters, and it records the Statures of 928 adult offspring of 205 Mid-Parents. It shows the distribution of Stature among the Sons of each successive group of Mid-Parents, in which the latter are all of the same Stature, reckoning to the nearest inch. I have calculated the M of each line, chiefly by drawing Schemes from the entries in it. Their values are printed at the ends of the lines and they form the right-hand column of the Table.

Tables 12 and 13 refer to the relation between Brothers. The one is derived from the R.F.F. and the other from the Special data. They both deal with small or moderately sized Fraternities, excluding the larger ones for reasons that will be explained directly, but the R.F.F. Table is the least restricted in this respect, as it only excludes families of 6 brothers and upwards. The data were so few in number that I could not well afford to lop off more. These Tables were constructed by registering the differences between each possible pair of brothers in each family: thus if there were three brothers, A, B, and C, in a particular family, I entered the differences of stature between A and B, A and C, and B and C., four brothers gave rise to 6 entries, and five brothers to 10 entries. The larger Fraternities were omitted, as the very large number of different pairs in them would have overwhelmed the influence of the smaller Fraternities. Large Fraternities are separately dealt with in Table 14.

We can derive some of the constants by more than one method; and it is gratifying to find how well the results of different methods confirm one another.

Mid-Stature of the Population.—The Median, Mid-Stature, or M of the general Population is a value of primary importance in this inquiry. Its value will be always designated by the symbol P, and it may be deduced from the bottom lines of any one of the three Tables. I obtain from them respectively the values 68·2, *68·5*, 68·4, but the middle of these, which is printed in italics, is a smoothed result. It is one of the only two smoothed values in the whole of my work, and was justifiably corrected, because the observed values that happen to lie nearest to the Grade of 50° ran out of harmony with the rest of the curve. It is therefore reasonable to consider its discrepancy as fortuitous, although it amounts to more than 0·15 inch. The series in question refers to R.F.F. brothers, who, owing to the principle on which the Table is constructed, are only a comparatively small sample taken out of the R.F.F. Population, and on a principle that gave greater weight to a few large families than to all the rest. Therefore it could not be expected to give rise to so regular a Scheme for the general R.F.F. Population as Table 11, which was fairly based upon the whole of it. Less accuracy was undoubtedly to have been expected in this group than in either of the others.

Variability of the Population.—The value of Q in the Statures of the general Population is to be deduced from the bottom lines of any one of the Tables 11, 12, and 13. The three values of it that I so obtain, are 1·65, 1·7, and 1·7 inch. I should mention that the method of the treatment originally adopted, happened also to make the first of these values 1·7 inch, so I have no hesitation in accepting 1·7 as the value for all my data.

Variability of Mid-Parents.—The value of Q in a Scheme drawn from the Statures of the R.F.F. Mid-Parents according to the data in Table 11, is 1·19 inches. Now it has already been shown that if marriage selection is independent of stature, the value of Q in the Scheme of Mid-parental Statures would be equal to its value in that of the general Population (which we have just seen to be 1·7 inch), divided by the square root of 2; that is by 1·45. This calculation makes it to be

1·21 inch, which agrees excellently with the observed value.[1]

Variability in Co-Fraternities.—As all the Adult Sons and Transmuted Daughters of the *same* Mid-Parent, form what is called a Fraternity, so all the Adult Sons and Transmuted Daughters of a *group* of Mid-Parents who have the same Stature (reckoned to the nearest inch) will be termed a Co-Fraternity. Each line in Table 11 refers to a separate Co-Fraternity and expresses the distribution of Stature among them. There are three reasons why Co-Fraternals should be more diverse among themselves than brothers. *First*, because their Mid-Parents are not of identical height, but may differ even as much as one inch. Secondly, because their grandparents, great-grandparents, and so on indefinitely backwards, may have differed widely. Thirdly, because the nurture or rearing of Co-Fraternals is more various than that of Fraternals. The brothers in a Fraternity of townsfolk do not seem to differ more among themselves than those in a Fraternity of country-folk, but a mixture of Fraternities derived indiscriminately from the two sources, must show greater diversity than either of them taken by themselves. The large differences between town and country-folk, and those between persons of different social classes, are conspicuous in the data contained in the Report of the

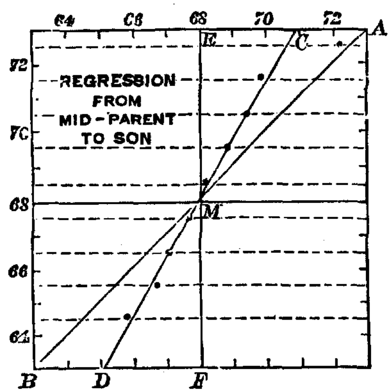
[1] In all my values referring to human stature, the second decimal is rudely approximate. I am obliged to use it, because if I worked only to tenths of an inch, sensible errors might creep in entirely owing to arithmetical operations.

Anthropological Committee to the British Association in 1880, and published in its Journal.

I concluded after carefully studying the chart upon which each of the individual observations from which Table 11 was constructed, had been entered separately in their appropriate places, and not clubbed into groups as in the Tables, that the value of Q in each Co-Fraternal group was roughly the same, whatever their Mid-Parental value might have been. It was not quite the same, being a trifle larger when the Mid-Parents were tall than when they were short. This justifies what will be said in Appendix E about the Geometric Mean; it also justifies neglect in the present inquiry of the method founded upon it, because the improvement in the results to which it might lead, would be insignificant, while its use would have added to the difficulty of explanation, and introduced extra trouble throughout, to the reader more than to myself. The value that I adopt for Q in every Co-Fraternal group, is 1·5 inch.

Regression.—a. Filial: However paradoxical it may appear at first sight, it is theoretically a necessary fact, and one that is clearly confirmed by observation, that the Stature of the adult offspring must on the whole, be more *mediocre* than the stature of their Parents; that is to say, more near to the M of the general Population. Table 11 enables us to compare the values of the M in different Co-Fraternal groups with the Statures of their respective Mid-Parents. Fig. 10 is a graphical representation of the meaning of the Table so far as it now concerns us. The horizontal dotted lines and the graduations at their sides, correspond to the similarly placed lines of figures and graduations in Table 11. The dot on each line shows the point where its M falls. The value of its M is to be read on the graduations along the top, and is the same as that which is given in the last column of Table 11. It will be perceived that the line drawn

FIG. 10.



through the centres of the dots, admits of being interpreted by the straight line C D, with but a small amount of give and take; and the fairness of this interpretation is confirmed by a study of the MS. chart above mentioned, in which the individual observations were plotted in their right places.

Now if we draw a line A B through every point where the graduations along the top of Fig. 10, are the same as those along the sides, the line will be straight and will run diagonally. It represents what the Mid-

Statures of the Sons would be, if they were on the average identical with those of their Mid-Parents. Most obviously A B does *not* agree with C D; therefore Sons do *not*, on the average, resemble their Mid-Parents. On examining these lines more closely, it will be observed that A B cuts C D at a point M that fairly corresponds to the value of 68$\frac{1}{4}$ inches, whether its value be read on the scale at the top or on that at the side. This is the value of P, the Mid-Stature of the population. Therefore it is only when the Parents are mediocre, that their Sons on the average resemble them.

Next draw a vertical line, E M F, through M, and let E C A be any horizontal line cutting ME at E, MC at E, and MA at A. Then it is obvious that the ratio of EA to EC is constant, whatever may be the position of E C A. This is true whether E C A be drawn above or like F D B, below M. In other words, the proportion between the Mid-Filial and the Mid-Parental deviation is constant, whatever the Mid-Parental stature may be. I reckon this ratio to be as 2 to 3: that is to say, the Filial deviation from P is on the average only two-thirds as wide as the Mid-Parental Deviation. I call this ratio of 2 to 3 the ratio of "Filial Regression." It is the proportion in which the Son is, on the average, less exceptional than his Mid-Parent.

My first estimate of the average proportion between the Mid-Filial and the Mid-Parental deviations, was made from a study of the MS. chart, and I then reckoned it as 3 to ·5. The value given above was afterwards substituted, because the data seemed to admit of that interpretation also, in which case the fraction of two-thirds was preferable as being the more simple expression. I am now inclined to think the latter may be a trifle too small, but it is not worth while to make alterations until a new, larger, and more accurate series of observations can be discussed, and the whole work revised. The present doubt only ranges between nine-fifteenths in the first case and ten-fifteenths in the second.

This value of two-thirds will therefore be accepted as the amount of Regression, on the average of many cases, from the Mid-Parental to the Mid-Filial stature, whatever the Mid-Parental stature may be.

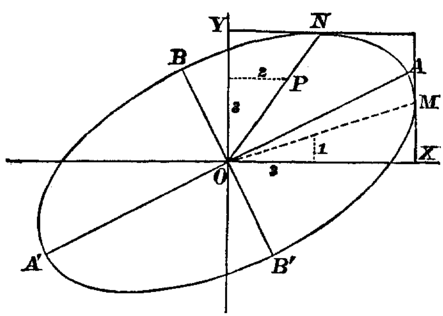
As the two Parents contribute equally, the contribution of either of them can be only one half of that *of the two jointly; in other words, only one half of that* of the Mid-Parent. Therefore the average Regression from the Parental to the Mid-Filial Stature must be the one half of two-thirds, or one-third. I am unable to test this conclusion in a satisfactory manner by direct observation. The data are barely numerous enough for dealing even with questions referring to Mid-Parentages; they are quite insufficient to deal with those that involve the additional large uncertainty introduced owing to an ignorance of the Stature of one of the parents. I have entered the Uni-Parental and the Filial data on a MS. chart, each in its appropriate place, but they are too scattered and irregular to make it useful to give the results in detail. They seem to show a Regression of about two-fifths, which differs from that of one-third in the ratio of 6 to 5. This direct observation is so inferior in value to the inferred result, that I disregard it, and am satisfied to adopt the value given by the latter, that is to say, of one-third, to express the average Regression from either of the Parents to the Son.

b. Mid-Parental: The converse relation to that which we have just discussed, namely the relation between the unknown stature of the Mid-Parent and the known Stature of the Son, is expressed by a fraction that is very far from being the converse of two-thirds. Though the Son deviates on the average from P only $\frac{2}{3}$ as widely as his Mid-parent, it does not in the least follow that the Mid-parent should deviate on the average from P, $\frac{3}{2}$ or $1\frac{1}{2}$, as widely as the Son. The Mid-Parent is not likely to be more exceptional than the son, but quite the contrary. The number of individuals who are nearly mediocre is so preponderant, that an exceptional man is more frequently found to be the exceptional son of mediocre parents than the average son of very exceptional parents. This is clearly shown by Table 11, where the very same observations which give the average value of Filial Regression when it is read in one way, gives that of the Mid-Parental Regression when it is read in another way, namely down the vertical columns, instead of along the horizontal lines. It then shows that the Mid-Parent of a man deviates on the average from P, only one-third as much as the man himself. This value of $\frac{1}{3}$ is four and a half times smaller than the numerical converse of $\frac{3}{2}$, since $4\frac{1}{2}$, or $\frac{9}{2}$, being multiplied into $\frac{1}{3}$, is equal to $\frac{3}{2}$.

c. Parental: As a Mid-Parental deviation is equal to one-half of the two Parental deviations, it follows that the Mid-Parental Regression must be equal to one-half of the sum of the two Parental Regressions. As the latter are equal to one another it follows that all three must have the same value. In other words, the average Mid-Parental Regression being $\frac{1}{3}$, the average Parental Regression must be $\frac{1}{3}$ also.

As there was much appearance of paradox in the above strongly contrasted results, I looked carefully into the run of the figures in Table 11. They were deduced, as already said, from a MS. chart on which the stature of every Son and the transmuted Stature of every Daughter is entered opposite to that of the Mid-Parent, the transmuted Statures being reckoned to the nearest tenth of an inch, and the position of the other entries being in every respect exactly as they were recorded. Then the number of entries in each square inch were counted, and copied in the form in which they appear in the Table. I found it hard at first to catch the full significance of the entries, though I soon discovered curious and apparently very interesting relations between them. These came out distinctly after I had "smoothed" the entries by writing at each intersection between a horizontal line and a vertical one, the sum of the entries in the four adjacent squares. I then noticed (see Fig. 11) that lines drawn through entries of the same value formed a series of concentric and similar ellipses. Their common centre lay at the intersection of those vertical and horizontal lines which correspond to the value of $68\frac{1}{4}$ inches, as read on both the top and on the side scales. Their axes were similarly inclined. The points where each successive ellipse was touched by a horizontal tangent, lay in a straight line that was inclined to the vertical in

FIG. 11.



the ratio of $\frac{2}{3}$, and those where the ellipses were touched by a vertical tangent, lay in a straight line inclined to the horizontal in the ratio of $\frac{1}{3}$. It will be obvious on studying Fig. 11 that the point where each successive horizontal line touches an ellipse is the point at which the greatest value in the line will be found. The same is true in respect to the successive vertical lines. Therefore these ratios confirm the values of the Ratios of Regression, already obtained by a different method, namely those of $\frac{2}{3}$ from Mid-Parent to Son, and of

$\frac{1}{3}$ from Son to Mid-Parent. These and other relations were evidently a subject for mathematical analysis and verification. It seemed clear to me that they all depended on three elementary measures, supposing the law of Frequency of Error to be applicable throughout; namely (1) the value of Q in the General Population, which was found to be 1·7 inch; (2) the value of Q in any Co-Fraternity, which was found to be 1·5 inch; (3) the Average Regression of the Stature of the Son from that of the Mid-Parent, which was found to be $\frac{2}{3}$. I wrote down these values, and phrasing the problem in abstract terms, disentangled from all reference to heredity, submitted it to Mr. J. D. Hamilton Dickson, Tutor of St. Peter's College, Cambridge (see Appendix B). I asked him kindly to investigate for me the Surface of Frequency of Error that would result from these three data, and the various shapes and other particulars of its sections that were made by horizontal planes, inasmuch as they ought to form the ellipses of which I spoke.

The problem may not be difficult to an accomplished mathematician, but I certainly never felt such a glow of loyalty and respect towards the sovereignty and wide sway of mathematical analysis as when his answer arrived, confirming, by purely mathematical reasoning, my various and laborious statistical conclusions with far more minuteness than I had dared to hope, because the data ran somewhat roughly, and I had to smooth them with tender caution. His calculation corrected my observed value of Mid-Parental Regression from $\frac{1}{3}$ to $\frac{6}{17.6}$; the relation between the major and minor axis of the ellipses was changed 3 per cent.; and their inclination to one another was changed less than 2°.[1]

It is obvious from this close accord of calculation with observation, that the law of Error holds throughout with sufficient precision to be of real service, and that the various results of my statistics are not casual and disconnected determinations, but strictly interdependent.

I trust it will have become clear even to the most non-mathematical reader, that the law of Regression in Stature refers primarily to Deviations, that is, to measurements made from the *level of mediocrity* to the

[1] The following is a more detailed comparison between the calculated and the observed results. The latter are enclosed in brackets. The letters refer to Fig. 11:—

Given—

The "Probable Error" of each system of Mid-Parentages = 1·22 inch. (This was an earlier determination of its value; as already said, the second decimal is to be considered only as approximate.)

Ratio of mean filial regression = $\frac{2}{3}$.

"Prob. Error" of each Co-Fraternity = 1·50 inch.

Sections of surface of frequency parallel to XY are true ellipses.
(Obs.—Apparently true ellipses.)
MX : YO = 6 : 17·5, or nearly 1 : 3.
(Obs.—1 : 3.)
Major axes to minor axes = $\sqrt{7} : \sqrt{2}$ = 10 : 5·35.
(Obs.—10 : 5·1.)
Inclination of major axes to OX = 26° 36′.
(Obs. 25°.)
Section of surface parallel to XZ is a true Curve of Frequency.
(Obs.—Apparently so.)
"Prob. Error", the Q of that curve, = 1.07 inch.
(Obs.—1·00, or a little more.)

crown of the head, upwards or downwards as the case may be, and not from the *ground* to the crown of the head. (In the population with which I am now dealing, the level of mediocrity is $68\frac{1}{4}$ inches (without shoes).) The law of Regression in respect to Stature may be phrased as follows; namely, that the Deviation of the Sons from P are, on the average, equal to one-third of the deviation of the Parent from P, and in the same direction. Or more briefly still:—If $P + (\pm \mathsf{D})$ be the Stature of the Parent, the Stature of the offspring will on the average be $P + (\pm \frac{1}{3} \mathsf{D})$.

If this remarkable law of Regression had been based only on those experiments with seeds, in which I first observed it, it might well be distrusted until otherwise confirmed. If it had been corroborated by a comparatively small number of observations on human stature, some hesitation might be expected before its truth could be recognised in opposition to the current belief that the child tends to resemble its parents. But more can be urged than this. It is easily to be shown that we ought to expect Filial Regression, and that it ought to amount to some constant fractional part of the value of the Mid-Parental deviation. All of this will be made clear in a subsequent section, when we shall discuss the cause of the curious statistical constancy in successive generations of a large population. In the meantime, two different reasons may be given for the occurrence of Regression; the one is connected with our notions of stability of type, and of which no more need now be said; the other is as follows:—The child inherits partly from his parents, partly from his ancestry. In every population that intermarries freely, when the genealogy of any man is traced far backwards, his ancestry will be found to consist of such varied elements that they are indistinguishable from a sample taken at haphazard from the general Population. The Mid-Stature M of the remote ancestry of such a man will become identical with P; in other words, it will be mediocre. To put the same conclusion into another form, the most probable value of the Deviation from P, of his Mid-Ancestors in any remote generation, is zero.

For the moment let us confine our attention to some one generation in the remote ancestry on the one hand, and to the Mid-Parent on the other, and ignore all other generations. The combination of the zero Deviation of the one with the observed Deviation of the other is the combination of nothing with something. Its effect resembles that of pouring a measure of water into a vessel of wine. The wine is diluted to a constant fraction of its alcoholic strength, whatever that strength may have been.

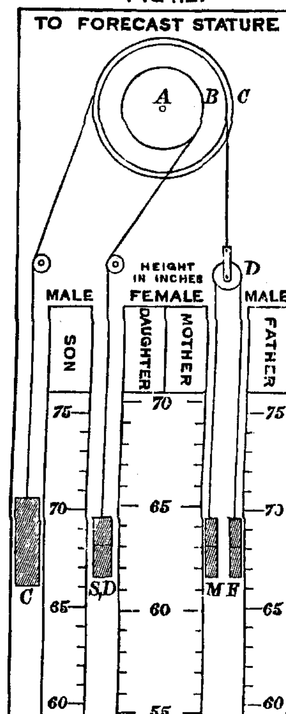
Similarly with regard to every other generation. The Mid-Deviation in any near generation of the ancestors will have a value intermediate between that of the zero Deviation of the remote ancestry, and of the observed Deviation of the Mid-Parent. Its combination with the Mid-Parental Deviation will be as if a mixture of wine and water in some definite proportion, and not pure water, had been poured into the wine. The process throughout is one of proportionate dilutions, and the joint effect of all of them is to weaken the original alcoholic strength in a constant ratio.

The law of Regression tells heavily against the full hereditary transmission of any gift. Only a few out of many children would be likely to differ from mediocrity so widely as *their* Mid-Parent, and *still* fewer would differ as widely as the more exceptional of the two Parents. The more bountifully the Parent is gifted by nature, the more rare will be his good fortune if he begets a son who is as richly endowed as himself, and still more so if he has a son who is endowed yet more largely. But the law is even-handed; it levies an equal succession-tax on the transmission of badness as of goodness. If it discourages the extravagant hopes of a gifted parent that his children will inherit all his powers; it no less discountenances extravagant fears that they will inherit all his weakness and disease.

It must be clearly understood that there is nothing in these statements to invalidate the general doctrine that the children of a gifted pair are much more likely to be gifted than the children of a mediocre pair. They merely express the fact that the ablest of all the children of a few gifted pairs is not likely to be as gifted as the ablest of all the children of a very great many mediocre pairs.

The constancy of the ratio of Regression, whatever may be the amount of the Mid-Parental Deviation, is now seen to be a reasonable law which might have been foreseen. It is so simple in its relations that I have contrived more than one form of apparatus by which the probable stature of the children of known parents can be mechanically reckoned. Fig. 12 is a representation of one of them, that is worked with pulleys and weights. A, B, and C are three thin wheels with grooves round their edges. They are screwed together so as to form a single piece that turns easily on its axis. The weights M and F are attached to either end of a thread that passes over the movable pulley D. The pulley itself hangs from a thread which is wrapped two or three times round the groove of B and is then secured to the wheel. The weight SD hangs from a thread that is wrapped two or three times round the groove of A, and is then secured to the wheel. The diameter of A is to that of B as 2 to 3. Lastly, a thread is wrapped in the opposite direction round the wheel C, which may have any convenient diameter, and is attached to a counterpoise. M refers to the male statures, F to the female ones, S to the Sons, D to the Daughters.

FIG. 12.



The scale of Female Statures differs from that of the Males, each Female height being laid down in the position which would be occupied by its male equivalent.

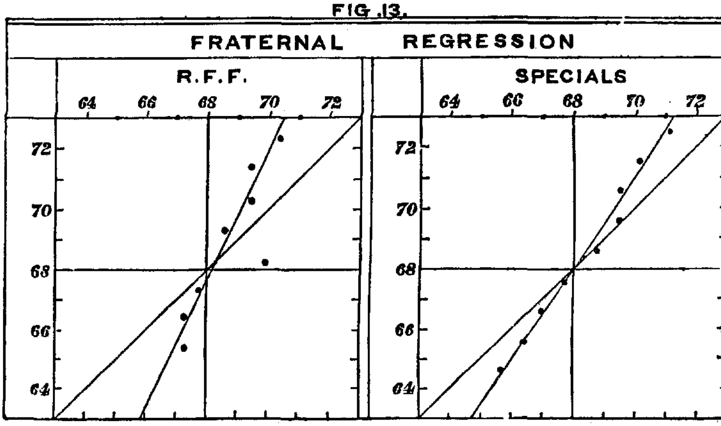
Thus 56 is written in the position of 60·48 inches, which is equal to 56 × 1·08. Similarly, 60 is written in the position of 64·80, which is equal to 60 × 1·08.

It is obvious that raising M will cause F to fall, and *vice versâ*, without affecting the wheel AB, and therefore without affecting SD; that is to say, the Parental Differences may be varied indefinitely without affecting the Stature of the children, so long as the Mid-Parental Stature is unchanged. But if the Mid-Parental Stature is changed to any specified amount, then that of SD will be changed to $\frac{2}{3}$ of that amount.

The weights M and F have to be set opposite to the heights of the mother and father on their respective scales; then the weight SD will show the most probable heights of a Son and of a Daughter on the corresponding scales. In every one of these cases, it is the fiducial mark in the middle of each weight by which the reading is to be made. But, in addition to this, the length of the weight SD is so arranged that it is an equal chance (an even bet) that the height of each Son or each Daughter will lie within the range defined by the upper and lower edge of the weight, on their respective scales. *The length of SD is 3 inches, which is twice the Q of* the Co-Fraternity; that is, 2 × 1·50 inch.

d. Fraternal: In seeking for the value of Fraternal Regression, it is better to confine ourselves to the Special data given in Table 13, as they are much more trustworthy than the R.F.F. data in Table 12. By treating them in the way shown in Fig. 13, which is constructed on the same principle as Fig. 10, page 96, I obtained the value for Fraternal Regression of $\frac{2}{3}$; that is to say, the unknown brother of a known man is probably only two-thirds as exceptional in Stature as he is. This is the same value as that obtained for the Regression from Mid-Parent to Son. However paradoxical the fact may seem at first, of there being such a thing as Fraternal Regression, a little reflection will show its reasonableness, which will become much clearer later on. In the meantime, we may recollect that the

FIG. 13.



unknown brother has two different tendencies, the one to resemble the known man, and the other to resemble his race. The one tendency is to deviate from P as *much as his brother, and the other tendency is not* to deviate at all. The result is a compromise.

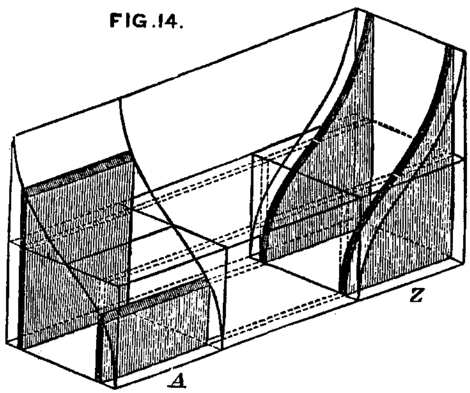
As the average Regression from either Parent to the Son is twice as great as that from a man to his Brother, a man is, generally speaking, only half as nearly related to either of his Parents as he is to his Brother. In other words, the Parental kinship is only half as close as the Fraternal.

We have now seen that there is Regression from the Parent to his Son, from the Son to his Parent, and from the Brother to his Brother. As these are the only three possible lines of kinship, namely, descending, ascending, and collateral, it must be a universal rule that the unknown Kinsman, in any degree, of a known Man, is on the average more mediocre than he. Let $P \pm D$ be the stature of the known man, and $P \pm D'$ the stature of his as yet unknown kinsman, then it is safe to wager, in the absence of all other knowledge, that D' is less than D.

Squadron of Statures.—It is an axiom of statistics, as I need hardly repeat, that every large sample taken at random out of any still larger group, may be considered as identical in its composition, in such inquiries as these in which we are now engaged, where minute accuracy is not desired and where highly exceptional cases are not regarded. Suppose our larger group to consist of a million, that is of 1000×1000 statures, and that we had divided it at random into 1000 samples each containing 1000 statures, and made Schemes of each of them. The 1000 Schemes would be practically identical, and we might marshal them one behind the other in successive ranks, and thereby form a "Squadron," numbering 1000 statures each way, and standing upon a square base. Our Squadron may be divided either into 1000 ranks or into 1000 files. The ranks will form a series of 1000 identical Schemes, the files will form a series of 1000 rectangles, that are of the same breadth, but of dissimilar heights. (See Fig. 14.)

It is easy by this illustration to give a general idea, to be developed as we proceed, of the way in which any large sample, A, of a Population gives rise to a group of Kinsmen, Z, so distant as to retain no family likeness

FIG. 14.



to A, but to be statistically undistinguishable from the Population generally, as regards the distribution of their statures. In this case the samples A and Z would form similar Schemes. I must suppose provisionally, for the purpose of easily arriving at an approximate theory, that tall, short, and mediocre Parents contribute equally to the next generation though this may not strictly be the case.[1]

[1] Oddly enough, the shortest couple on my list have the largest family, namely, sixteen children, of whom fourteen were measured.

Throw A into the form of a Squadron and not of a Scheme, and let us begin by confining our attention to the men who form any two of the rectangular files of A, that we please to select. Then let us trace their connections with their respective Kinsmen in Z. As the number of the Z Kinsmen to each of the A files is *considered to be the same, and as their respective* Stature-Schemes are supposed to be identical with that of the general Population, it follows that the two Schemes in Z derived from the two different rectangular files in A, will be identical with one another. Every other rectangular file in A will be similarly represented by another identical Scheme in Z. Therefore the 1,000 different rectangular files in A will produce 1,000 identical Schemes in Z, arranged as in Fig. 14.

Though all the Schemes in Z, contain the same number of measures, each will contain many more measures than were contained in the files of A, because the same kinsmen would usually be counted many times over. Thus a man may be counted as uncle to many nephews, and as nephew to many uncles. We will therefore (though it is hardly necessary to do so) suppose each of the files in Z to have been constructed from only a sample consisting of 1,000 persons, taken at random out of the more numerous measures to which it refers. By this treatment Z becomes an exact Squadron, consisting of 1,000 elements, both in rank and in file, and it is identical with A in its constitution, though not in its attitude. The ranks of Z, which are Schemes, have been derived from the files of A, which are rectangles, therefore the two Squadrons must stand at right angles to one another, as in Fig. 14. The upper surface of A is curved in rank, and horizontal in file; that of Z is curved in file and horizontal in rank.

The Kinsmen in nearer degrees than Z will be represented by Squadrons whose forms are intermediate between A and Z. Front views of these are shown in

FIG. 15.

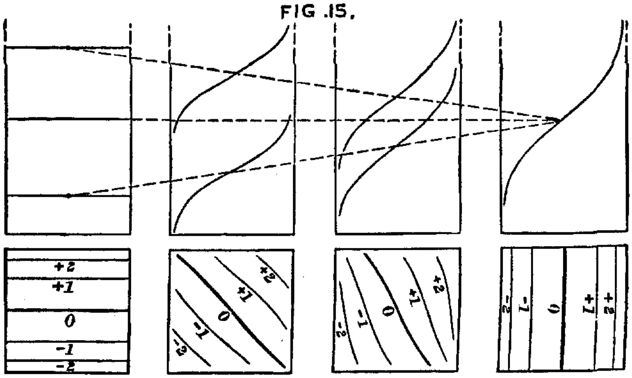


Fig. 15. Consequently they will be somewhat curved both in rank and in file. Also as the Kinsmen of all the members of a Population, in any degree, are themselves a Population having similar characteristics to those of the Population of which they are part, it follows that the elements of every intermediate Squadron when they are broken up and sorted afresh into ordinary Schemes, would form identical Schemes. Therefore, it is clear that a law exists that connects the curvatures in rank and in file, of any Squadron. Both of the curvatures are Curves of Distribution; let us call their Q values respectively r and f. Then if p be the Q of

the general Population, we arrive at a general equation that is true for all degrees of Kinship; namely—

$$r^2 + f^2 = p^2 \qquad (1)$$

but r, the curvature in rank, is a regressed value of p, and may be written wp, w being the value of the Regression. Therefore the above equation may be put in the form of

$$w^2p^2 + f^2 = p^2 \qquad (2)$$

in which f is the Q of the Co-kinsmen in the given degree.

It will be found that the intersection of the surfaces of the Squadrons by a horizontal plane, whose height is equal to P, forms in each case a line, whose general inclination to the ranks of A increases as the Kinship becomes more remote, until it becomes a right angle in Z. The progressive change of inclination is shown in the small squares drawn at the base of Fig. 13, in which the lines are the projections of contours drawn on the upper surfaces of the Squadrons, to correspond with the multiples there stated of values of p.

It will be understood from the front views of the four different Squadrons, which form the upper part of Fig. 13, how the Mid-Statures of the Kinsmen to the Men in each of the files of A, gradually become more mediocre in the successive stages of kinship until they all reach absolute mediocrity in Z. This figure affords an excellent diagramatic representation, true to scale, of the action of the law of Regression in Descent. I should like to have given in addition, a perspective view of the Squadrons, but failed to draw them clearly, after making many attempts. Their curvatures are so delicate and peculiar that the eye can hardly appreciate them even in a model, without turning it about in different lights and aspects. A plaster model of an intermediate form was exhibited at the Royal Society by Mr. J. D. H. Dickson, when my paper on *Hereditary Stature* was read, together with his solutions of the problems that are given in the Appendix. I also exhibited arrangements of files and ranks that were made of pasteboard. Mr. Dixon mentioned that the mathematical properties of a Surface of Frequency showed that no strictly straight line could be drawn upon it.

Successive Generations of a People.—We are far too apt to regard common events as matters of course, that require no explanation, whereas they may be problems of much interest and of some difficulty, and still await solution.

Why is it, when we compare two large groups of persons selected at random from the same race, but belonging to different generations, that they are usually found to be closely alike? There may be some small statistical dissimilarity due to well understood differences in the general conditions of their lives, but with this I am not concerned. The present question is as to the origin of that statistical resemblance between successive generations which is due to the strict processes of heredity, and which is commonly observed in all forms of life.

In each generation, individuals are found to be tall and short, heavy and light, strong and weak, dark and pale; and the proportions of those who present these several characteristics in their various degrees, tend to be constant. The records of geological history afford striking evidences of this statistical similarity. Fossil remains of plants and animals may be dug out of strata at such different levels, that thousands of generations must have intervened between the periods at which they lived; yet in large samples of such fossils we may seek in vain for peculiarities that distinguish one generation from another, the different sizes, marks, and variations of every kind, occurring with equal frequency in all.

If any are inclined to reply that there is no wonder in the matter, because each individual tends to leave his like behind him, and therefore each generation must, as a matter of course, resemble the one preceding, the patent fact of Regression shows that they utterly misunderstand the case.

We have now reached a stage at which it has become possible to discuss the problem with some exactness, and I will do so by giving mathematical expression to what actually took place in the Statures of that sample of our Population whose life-histories are recorded in the R.F.F. data.

The Males and Females in Generation I. whose **M** has the value of P (viz., $68\frac{1}{4}$ inches), and whose **Q** is 1·7 inch, were found to group themselves as it were at random, into couples, and then to form a system of

Mid-Parents. This system had of course the same M as the general Population, but its Q was reduced to $\frac{1}{\sqrt{2}} \times 1 \cdot 7$ inch, or to 1·2 inch. It was next found when the Statures of the Mid-Parents, expressed in the form of $P + (\pm D)$, were sorted into groups in which D was the same (reckoning to the nearest inch), that a Co-fraternity sprang from each group, and that its M had the value of $P + (\pm \frac{2}{3} D)$. The system in which each element is a Mid-Co-Fraternity, must have the same M as before, of $68\frac{1}{4}$ inches, but its Q will be again reduced, namely from 1·2 inch to $\frac{2}{3} \times 1 \cdot 2$ inch, or to 0·8 inch. Lastly, the individual Co-Fraternals were seen to be dispersed from their respective Mid-Co-Fraternities, with a Q equal in each case to 1·5 inch. The sum of all of the Co-Fraternals forms the Population of Generation II. Consequently the members of Generation II. constitute a system that has an M of $68\frac{1}{4}$ inches and a Q equal to $\sqrt{[(0 \cdot 8)^2 + (1 \cdot 5)^2]}$, $= 1 \cdot 7$ inch. These values are identical with those in Generation I.; so the cause of their statistical similarity is tracked out.

There ought to be no misunderstanding as to the character of the evidence or of the reasoning upon which this analysis is based. A small but fair sample of the Population in two successive Generations has been taken, and its conditions as regards Stature have been strictly discussed. It was found that the distribution of Stature was sufficiently Normal to justify our ignoring any shortcomings in that respect. The transmutation of female heights to their male equivalents was justified by the fact that when the individual Statures of a group of females are raised in the proportion of 100 to 108, the Scheme drawn from them fairly coincides with that drawn from male Statures. Marriage selection was found to take no sufficient notice of Stature to be worth consideration ; neither was the number of children in Fraternities found to be sensibly affected by the Statures of their Parents. Again, it was seen to be of no consequence when dealing statistically with the offspring, whether their Parents were alike in stature or not, the only datum deserving consideration being the Stature of the Mid-Parent, that is to say, the average value of (1) the Stature of the Father, and of (2) the Transmuted Stature of the Mother. I fully grant that not one of these deductions may be strictly exact, but the error introduced into the conclusions by supposing them to be correct proves not to be worth taking into account in a first approximation.

Precisely the same may be said of the ulterior steps in this analysis. Every one of them is based on the properties of an ideally perfect curve, but in no case has there been need to make any sensible departure from the observed results, except in assigning a uniform value to Q in the different Co-Fraternities. Strictly speaking, that value was found to slightly rise or fall as the Mid-Stature of the Co-Fraternity rose or fell. This suggested the advisability of treating the whole inquiry on the principle of the Geometric Mean, Appendix G. I tried that principle in what seemed to be the most hopeful case among my 18 schemes, but found the gain, if any, to be so small, that I did not care to go on with the experiment. It did not seem to deserve the additional trouble, and I was indisposed to do anything that was not really necessary, which might further confuse the reader. But had I possessed better data, I should have tried the Geometric Mean throughout. In doing so, every measure would be replaced by its logarithm, and these logarithms would be treated just as if they had been the observed values. The conclusions to which they might lead would then be re-transmuted to the numbers of which they were the logarithmic equivalents.

In short, we have dealt mathematically with an ideal population which has similar characteristics to those of a real population, and have seen how closely the behaviour of the ideal population corresponds in every stage to that of the real one. Therefore we have arrived at a closely approximate solution of the problem of statistical constancy, though numerous refinements have been neglected.

Natural Selection.—This hardly falls within the scope of our inquiry into Natural Inheritance, but it will be appropriate to consider briefly the way in which the action of Natural Selection may harmonise with that of pure heredity, and work together with it in such a manner as not to compromise the normal distribution of faculty. To do this, we must deal with the case that best represents the various possible occurrences, namely that in which the mediocre members of a population are those that are most nearly in harmony with their circumstances. The harmony ought to concern the aggregate of their faculties, combined on the principle adopted in Table 3, after weighting them in the order of their importance. We may deal with any faculty separately, to serve as an example, if its mediocre value happens to be that which is most preservative of life under the majority of circumstances. Such is Stature, in a rudely approximate degree, inasmuch as exceptionally tall or exceptionally short persons have less chance of life than those of moderate size.

It will give more definiteness to the reasoning to take a definite example, even though it be in part an imaginary one. Suppose then, that we are considering the stature of some animal that is liable to be hunted by certain beasts of prey in a particular country. So far as he is big of his kind, he would be better able than the mediocrities to crush through thick grass and foliage whenever he was scampering for his life, to jump over obstacles, and possibly to run somewhat faster than they. So far as he is small of his kind, he would be better able to run through narrow openings, to make quick turns, and to hide himself. Under the general circumstances, it would be found that animals of some particular stature had on the whole a better chance of escape than any other, and if their race is closely adapted to their circumstances in respect to stature, the most favoured stature would be identical with the M of the race. We already know that if we call this value P, and write each stature under the form of $P+x$ (in which x includes its sign), and if the number of times with which any value $P+x$ occurs, compared to the number of times in which P occurs, be called y, then x and y are connected by the law of Frequency of Error.

Though the impediments to flight are less unfavourable, on the average, to the stature P than to any other, they will differ in different experiences. The course of one animal may chance to pass through denser foliage than usual, or the obstacles in his way may be higher. In that case an animal whose stature exceeded P would have an advantage over mediocrities. Conversely, the circumstances might be more favourable to a small animal.

Each particular line of escape would be most favourable to some particular stature, and whatever the value of x might be, it is possible that the stature $P+x$ might in some cases be more favoured than any other. But the accidents of foliage and soil in a country are characteristic and persistent, and may fairly be considered as approximately of a typical kind. Therefore those that most favour the animals whose stature is P will be more frequently met with than those that favour any other stature $P+x$, and the frequency of the latter occurrence will diminish rapidly as x increases. If the number of times with which any particular value of $P+x$ is most favoured, as compared with the number of times in which P is most favoured, be called y', we may fairly assume that y' and x are connected by the law of Frequency of Error. But though the system of y values and that of y' values may be both subject to the law, it is not for a moment to be supposed that their Q values are necessarily the same.

We have now to show how a large population of animals becomes reduced by the action of natural selection to a smaller one, in which the M value of the statures is unchanged, while the Q value is decreased.

To do this we must first consider the population to have grown up entirely shielded from causes of premature mortality; call their number N. Then suppose them to be assailed by all the lethal influences that have no reference to stature. These would reduce their number to N′, but by the hypothesis, the values of M and of Q would remain unaffected. Next let the influences that act selectively on stature, further reduce the numbers to S; these being the final survivors. We have seen that:—

$y=$ the number of individuals who have the stature $P\pm x$, counting those who have the stature P, as 1.

$y'=$ the number of times in which $P\pm x$ is the most favoured stature, counting those in which P is the most favoured, as 1.

Then $yy'=$ the number of times that individuals of the stature $P\pm x$ are selected, counting those in which individuals of the stature P are selected, as 1.

As the relation between y and x, and between y' and x are severally governed by the law of Frequency of Error, it follows directly from the formula by which that law is expressed, that the relation between yy' and x is also governed by it. The value of P of course remains the same throughout, but the Q in the system of yy' values is necessarily less than that in the system of y values.

It might well be that natural selection would favour the indefinite increase of numerous separate faculties, if their improvement could be effected without detriment to the rest; then, mediocrity in that faculty would not be the safest condition. Thus an increase of fleetness would be a clear gain to an animal liable to be hunted by beasts of prey, if no other useful faculty was thereby diminished.

But a too free use of this "if" would show a jaunty disregard of a real difficulty. Organisms are so knit together that change in one direction involves change in many others; these may not attract attention, but they are none the less existent. Organisms are like ships of war constructed for a particular purpose in warfare, as cruisers, line of battle ships, &c., on the principle of obtaining the utmost efficiency for their special purpose. The result is a compromise between a variety of conflicting desiderata, such as cost, speed, accommodation, stability, weight of guns, thickness of armour, quick steering power, and so on. It is hardly possible in a ship of any long established type to make an improvement in any one of these respects, without a sacrifice in other directions. If the fleetness is increased, the engines must be larger, and more space must be given up to coal, and this diminishes the remaining accommodation. Evolution may produce an altogether new type of vessel that shall be more efficient than the old one, but when a particular type of vessel has become adapted to its functions through long experience it is not possible to produce a mere variety of its type that shall have increased efficiency in some one particular without detriment to the rest. So it is with animals.

Variability in Fraternities.—Human Fraternities are far too small to admit of their Q being satisfactorily measured by the direct method. We are obliged to have recourse to indirect methods, of which no less than four are available. I shall apply each of them to both the Special and to the R.F.F. data; this will give 8 separate estimates of its value, which in the meantime will be called *b*. The four methods are as follow:

First method; by Fraternities each containing the same number of persons:—Let me begin by saying that I had already found in the large Fraternities of Sweet Peas, that the sizes of individuals of whom they consisted were normally distributed, and that their Q was independent of the size, or of the Stature as we may phrase it, of the Mid-Fraternity. We have also seen that the Q is practically the same in all Co-fraternities of men. Therefore it is reasonable to expect that it will also be found to be the same in all their Fraternities, though owing to their small size we cannot assure ourselves of the fact by direct evidence. We will assume this to be the case for the present; it will be seen that the results justify the assumption.

The value of the M of a small Fraternity may be much affected by the addition or subtraction even of a single member, it may therefore be called the *apparent* M, to be distinguished from the *true* M, from which its members would be found to be dispersed, if there had been many more of them. The apparent M approximates towards the true M as the Fraternity increases in size, though at a much slower rate. We have now somehow to get at this true M. For distinction and for brevity let us call the *apparent* M of any small Fraternity (MF′), and that of the corresponding *true* M (MF). Then (MF) may be deduced from (MF′) as follows :—

We will begin by allowing ourselves for the moment to imagine the existence of an exceedingly large Fraternity, far more numerous than is physiologically possible, and to suppose that its members vary among themselves just as widely, neither more nor less so, than in the small Fraternities of real life. *The (MF′) of our large* ideal Fraternity will therefore be identical with its (MF), and its Q will be the same as b. Next, take at random out of this huge ideal Fraternity a large number of small samples, each consisting of the same number, n, of brothers, and call the apparent Mid-values in the several samples, (MF'_1), (MF'_2), &c. It can easily be shown that (MF'_1), (MF'_2), &c., will be so distributed about the common centre of (MF), that the Prob. Deviation of any one of them from it, that is to say, the Q of their system will $= b \times \frac{1}{\sqrt{n}}$. If $n = 1$, then the Prob. Deviation becomes b, as it should. If $n = 2$, the Prob.

Deviation is determined by the same problem as that which concerned the Q of the Mid-Parentages (page 87), where it was shown to be $b \times \frac{1}{\sqrt{2}}$. By similar reasoning, when $n = 3$, the Prob. Deviation becomes $b \times \frac{1}{\sqrt{3}}$, and so on. When n is infinitely large, the Prob. Deviation is 0; that is to say, the (MF′) values do not differ at all from their common (MF).

Now if we make a collection of human Fraternities, each consisting of the same number, n, of brothers, and note the differences between the (MF′) in each fraternity and the individual brothers, we shall obtain a system of values. By drawing a Scheme from these in the usual way, we are able to find their Q; let us call it d. We then determine b in terms of d, as follows:— The (MF′) values are distributed about their common (MF), each with the Prob. Deviation of $b \times \frac{1}{\sqrt{n}}$, and the Statures of the individual Brothers are distributed about their respective (MF′) values, each with the Prob. Deviation d. The compound result is the same as if the statures of the individual brothers had been distributed about the common (MF), each with the Prob. Deviation b,

$$\text{consequently } b^2 = d^2 + \frac{b^2}{n}, \quad \text{or } b^2 = \frac{n}{n-1} d^2.$$

I determined d by observation for four different values of n, having taken four groups of Fraternities, consisting respectively of 4, 5, 6, and 7 brothers, as shown in Table 14. Substituting these four observed values in turns for d in the above formula, I obtained four independent values of b, which are respectively 1·01, 1·01, 1·20, and 1·08; the mean of these is 1·07.

Second Method; from the mean value of Fraternal Regression:—We may look on the Population as composed of a system of Fraternities. Call their respective true centres (see last paragraph) (MF_1), (MF_2), &c. These will be distributed about P with an as yet unknown Prob. Deviation, that we will call c. The individual members of each Fraternity will of course be distributed from their own (MF) with a Q equal to b.

Then $(1{\cdot}7)^2 = c^2 + b^2$ (1)

Let $P + (\pm F_n)$ be the stature of any individual, and let $P + (\pm M\ F_n)$ be that of the **M** of his Fraternity, then Problem 4 (page 69) shows us that:—

the *most probable* value of $\frac{(MF_n)}{F_n}$ is $\sqrt{\frac{c^2}{b^2 + c^2}}$ (2)

This is also the value of Fraternal Regression, and therefore equal to $\frac{2}{3}$. Substituting in (2), and replacing c by the value given by (1), we obtain $b = 0{\cdot}98$ inch.

Third Method; by the Variability in the value of individual cases of Fraternal Regression:—The figures in each line of Table 13 are found to have a Q equal to 1·24 inch, and they are the results of two independent systems of variation. First, the several (MF) values (see last paragraph) are dispersed from the **M** of all of them with a Q that we will call v. Secondly the individual brothers in each Fraternity are dispersed from their own (MF) with a Q equal to b.

Hence $(1{\cdot}24)^2 = v^2 + b^2$.

But it is shown Problem 5 that $v = \dfrac{bc}{\sqrt{(b^2+c^2)}}$;

therefore $(1{\cdot}24)^2 = b^2 + \dfrac{b^2c^2}{b^2+c^2}$.

Substituting for c^2 its value of $(1{\cdot}7)^2 - b^2$ (see last paragraph), we obtain $b = 0{\cdot}98$ inch.

Fourth Method; from differences between pairs of brothers taken at random:—In the fourth method, Pairs of Brothers are taken at random, and the Differences between the statures in each pair are noted; then, under the following reservation, any one of these differences would have the Prob. value of $\sqrt{2} \times b$. The reservation is, that only as many Differences should be taken out of each Fraternity as are independent. A Fraternity of n brothers admits of $\frac{n(n-1)}{2}$ possible pairs, and the same number of Differences; but as no more than $n-1$ of these are independent, that number only of the Differences should be taken. I did not appreciate this necessity at first, and selected pairs of brothers on an arbitrary system, which had at all events the merit of not taking more than four sets of Differences from any one Fraternity however large it might be. It was faulty in taking three Differences instead of only two, out of a Fraternity of three brothers, and four Differences, instead of only three, from a Fraternity of four brothers, and therefore giving an increased weight to those Fraternities, but in other respects the system was hardly objectionable. The introduced error must be so slight as to make it scarcely worth while now to go over the work again. By the system adopted, I found the Prob. Difference to be 1·55, which divided by $\sqrt{2}$ gives b = 1·10 inch.

Thus far we have dealt with the special data only. The less trustworthy R.F.F. give larger values of b in every case. An epitome of all the results appears in the following table :—

Methods and data.	Values of b obtained by different methods and from different data.	
	From Special Data.	From R.F.F. data.[1]
(1) From Fraternities each containing the same number of persons	*1·07*	*1·38*
(2) From the mean value of Fraternal Regression............	0·98	1·31
(3) From the Variability of Fraternal Regression......... ..	1·10	1·14
(4) From Pairs of Brothers taken at random.................	1·10	1·35
Mean...............	*1·06*	

The data used in the four methods are somewhat different. In (1) I could not deal with small Fraterni-

[1] The R.F.F. results were obtained from brothers only and not from transmuted sisters, except in method (2), where the paucity of the data compelled me to include them.

ties, so all were disregarded that contained fewer than four individuals. In (2) and (3) I could not with safety use large Fraternities. In (4) the method of selection was, as we have seen, quite indifferent. This makes the accordance of the results derived from the Special data all the more gratifying. Those from the R.F.F. data accord less well together. The R.F.F. measures are not sufficiently exact for use in these delicate calculations. Their results, being compounded of b and of their tendency to deviate from exactness, are necessarily too high, and should be discarded. I gather from all this that we may safely consider the value of b to be less than 1·06, and that allowing for some want of precision in the Special data, the very convenient value of 1·00 inch may reasonably be adopted.

Trustworthiness of the Constants.—There is difficulty in correcting the results obtained from the R.F.F. data, though we can make some estimate of their general inaccuracy as compared with the Special data. The reason of the difficulty is that the inaccuracy cannot be ascribed to an uncertainty of equal $\pm$ amount in every entry, such as might be due to a doubt of "shoes off" or "shoes on." If it were so, the Prob. Error of a single value of the R.F.F. would be greater than that of one of the Specials, whereas it proves to be the same. It is likely that the inaccuracy is a compound first of the uncertainty above mentioned, whose effect would be to increase the value of the Prob. Error, and secondly of a tendency on the part of my correspondents to record medium statures when they were in doubt, whose effect would be to reduce the value of the Prob. Error. The R.F.F. data in Table 12 run so irregularly that I cannot interpret them with any assurance. The value they give for Fraternal Regression certainly does not exceed $\frac{1}{2}$, and therefore a correction, amounting to no less than $\frac{1}{3}$ of its amount, is required to bring it to a parity with that derived from the Special data (because $\frac{1}{2} + \frac{1}{3} \times \frac{1}{2} = \frac{2}{3}$). Hence it might be argued, that the value of Regression from Mid-Parent to Son, which the R.F.F. data gave as $\frac{2}{3}$, ought to receive a similar correction. If so, it would be raised to $\frac{2}{3} + \frac{2}{9} = \frac{8}{9}$; but I cannot believe this high value to be correct. My first estimate made from the R.F.F. data, was $\frac{3}{5}$, as already mentioned. If this be adopted, the corrected value would be $\frac{4}{5}$, or $\frac{8}{10}$ instead of $\frac{8}{9}$, which might possibly pass. Curiously enough, this value of $\frac{4}{5}$ for Regression from Mid-Parent to Son, coincides with the value of $\frac{2}{5}$ for Regression from a single Parent to Son, which the direct observations showed (see page 99), but which owing to their paucity and to the irregularity of the way in which they ran, I rejected and still reject, at least for the present. While sincerely desirous of obtaining a revised value of average Filial Regression from entirely different and more accurate groups of data, the provisional value already adopted of $\frac{2}{3}$ from Mid-Parent to Son may be accepted as being near enough for the present. It is impossible to revise one datum in the

R.F.F. series without revising all, as they hang together and support one another.

General View of Kinship.—We are now able to deal with the distribution of statures among the Kinsmen in every near degree, of persons whose statures we know, but whose ancestral statures we either do not know, or do not care to take into account. We are able to calculate Tables for every near degree of Kinship on the form of Table 11, and to reconstruct that same Table in a shape free from irregularities. We must first find the Regression, which we may call w, appropriate to the degree of Kinship in question. Then we calculate a value f for each line of a Table corresponding in form to that of Table 11, in which f was found to be equal to 1·50 inch. We deduce the value of f from that of w by means of the general equation $p^2w^2+f^2=p^2$, p being equal to 1·7 inch. The values to be inserted in the several lines are then calculated from the ordinary table (Table 5) of the "probability integral."

As an example of the first part of the process, let us suppose we are about to construct a table of Uncles and their Nephews, we find w and f as follows: A Nephew is the son of a Brother, therefore in this case we have $w=\frac{1}{3}\times\frac{2}{3}=\frac{2}{9}$; whence $f=1{\cdot}66$.

The Regression, which we call w, is a convenient and correct measure of family likeness. If the resemblance of the Kinsman to the Man, was on the average as perfect as that of the Man to his own Self, there would be no Regression at all, and the value of w would be 1.

TABLE OF DATA FOR CALCULATING TABLES OF DISTRIBUTION OF STATURE AMONG THE KINSMEN OF PERSONS WHOSE STATURE IS KNOWN.

From group of persons of the same Stature, to their Kinsmen in various near degrees.	Mean regression $=w$.	$Q=f$ $=p\times\sqrt{(1-w^2)}$.
Mid-parents to Sons	2 / 3	1·27
Brothers to Brothers	2 / 3	1·27
Fathers or Sons to Sons or Fathers	1 / 3	1·60
Uncles or Nephews to Nephews or Uncles	2 / 9	1·66
Grandsons to Grandparents...	1 / 9	Practically that of Population, or 1·7 inch.
Cousins to Cousins	2 / 27	

On the other hand, if the Kinsmen were on the average no more like the Man than if they had been a group picked at random out of the general Population, then the Regression to P would be complete. The M of the Kinsmen, which is expressed by $P+w(\pm D)$, would in that case become P, whatever might have been the value of D; therefore w must$=0$. We see by the preceding Table that as a general rule, Fathers or Sons should be held to be only one-half as near in blood as Brothers, and Uncles and Nephews to be one-third as near in blood as Brothers. Cousins are $4\frac{1}{2}$ times as remote as Fathers or as Sons, and 9 times as remote as Brothers. I do not extend the table further, because considerations would have to be taken into account that will be discussed in the next Section.

The remarks made in a previous chapter about heritages that blend and those that are mutually exclusive, must be here borne in mind. It would be a poor prerogative to inherit say the fifth part of the peculiarity of some gifted ancestor, but the chance of 1 to 5, of inheriting the whole of it, would be deservedly prized.

Separate Contribution of each Ancestor.—In making the statement that Mid-Parents whose Stature is $P \pm D$ have children whose average stature is $P \pm \frac{2}{3}D$, it is supposed that no separate account has been taken of the previous ancestry. Yet though nothing may be known of them, something is tacitly implied and has been tacitly allowed for, and this requires to be eliminated before we can learn the amount of the Parental bequest, pure and simple. What that something is, we must now try to discover. When speaking of converse Regression, it was shown that a peculiarity in a Man implied a peculiarity of $\frac{1}{3}$ of that amount in his Mid-Parent. Call the peculiarity of the Mid-Parent D, then the implied peculiarity of the Mid-Parent of the Mid-Parent, that is of the Mid-Grand-Parent of the Man, would on the above supposition be $\frac{1}{3}D$, that of the Mid-Great-Grand-Parent would be $\frac{1}{9}D$, and so on. Hence the total bequeathable property would amount to $D(1 + \frac{1}{3} + \frac{1}{9} + \&c.) = D\frac{3}{2}$.

Do the bequests from each of the successive generations reach the child without any, or what, diminution by the way? I have not sufficient data to yield a direct reply, and must therefore try limiting suppositions.

First, suppose the bequests by the various generations to be equally taxed; then, as an accumulation of ancestral contributions whose sum amounts to D$\frac{3}{2}$ yields an effective heritage of only D$\frac{2}{3}$, it follows that each piece of heritable property must have been reduced to $\frac{4}{9}$ of its original amount, because $\frac{3}{2} \times \frac{4}{9} = \frac{2}{3}$.

Secondly, suppose the tax not to be uniform, but to be repeated at each successive transmission, and to be equal to $\frac{1}{r}$ of the amount of the property at each stage. In this case the effective heritage would be $D\left(\frac{1}{r}+\frac{1}{3r^2}+\frac{1}{3^2r^3}+ -\right)=D\,\frac{3}{3r-1}$, which must, as before, be equal to D$\frac{2}{3}$; whence $\frac{1}{r}=\frac{6}{11}$

Thirdly, it might possibly be supposed that the Mid-Ancestor in a remote generation should on the average contribute more to the child than the Mid-Parent, but this is quite contrary to what is observed. The descendants of what was "pedigree wheat," after being left to themselves for many generations, show little or no trace of the remarkable size of their Mid-Ancestors in the generations just before they were left to themselves, though the offspring of those Mid-Ancestors in the first generation did so unmistakably.

The results of our only two valid limiting suppositions are therefore, (1) that the Mid-Parental peculiarities, pure and simple, influence the offspring to $\frac{4}{9}$ of their amount; (2) that they influence it to $\frac{6}{11}$ of their amount. These values differ but slightly from $\frac{1}{2}$, and their mean is closely $\frac{1}{2}$, so we may fairly accept that result. Hence the influence, pure and simple, of the Mid-Parent may be taken as $\frac{1}{2}$, and that of the Mid-Grand-Parent as $\frac{1}{4}$, and so on. Consequently the influence of the individual Parent would be $\frac{1}{4}$, and of the individual Grand-Parent $\frac{1}{16}$, and so on. It would, however, be hazardous on the present slender basis, to extend this sequence with confidence to more distant generations.

Pedigree Moths.—I am endeavouring at this moment to obtain data that will enable me to go further, by breeding Pedigree Moths, thanks to the aid of Mr. Frederick Merrifield. The moths *Selenia Illustraria* and *Illunaria* are chosen for the purpose, partly on account of their being what is called double brooded; that is to say, they pass normally through two generations in a single year, which is a great saving of time to the experimenter. They are hardy, prolific and variable, and are found to stand chloroform well, previously to being measured and then paired. Every member of each Fraternity is preserved along three lines of descent—one race of long-winged moths, one of medium-winged, and one of short-winged moths. The three parallel sets are reared under identical conditions, so that the medium series supplies a trustworthy relative base, from which to measure the increasing divergency of the others. No one can be sure of the success of any extensive breeding experiment, but this attempt has been well started and seems to present no peculiar difficulty. Among other reasons for choosing moths for the purpose, is that they are born adults, not changing in stature after they have emerged from the chrysalis and shaken out their wings. Their families are of a convenient size for statistical purposes, say from 50 to 100, neither too few to make satisfactory Schemes, nor unmanageably large. They can be mounted as we all know, after their death, with great facility, and be remeasured at leisure. An intelligent and experienced person can carry on a large breeding establishment in a small room, supplemented by a small garden. The methods used and the results up to last spring, have been described by Mr. Merrifield in papers read February and December 1887, and printed in the Transactions of the Entomological Society. I speak of this now, in hopes of attracting the attention of some who are competent and willing to carry on collateral experiments with the same breed, or with altogether different species of moths.